Gravitational interaction of nucleons with mini black holes

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We analyzed the interaction of the nonreltivistic nucleons with mini black holes (MBH). MBH are hypothetical objects having enormous mass density. The mass of the MBH (in g) can be estimated from equation

$$M = 6.74 \times 10^{27} R \; g\,, \tag{1}$$

where its radius R is expressed in cm. For example, for $R = 10^{-13}$ we get $M = 6.74\times 10^{14}$ g. Such a gigantic mass curves the space around. In this work we consider the gravitational interaction of the MBHs with non-relativistic nucleons. Evidently that requires derivation of the equation for the wave function in a curved space. To do it we start from the Klein-Gordon equation, which is written in the covariant form in the general relativity:

$$\hbar^2 \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \sqrt{-g}\, g^{\mu\nu} \frac{\partial}{\partial x^\nu} \Psi = -m^2 c^2 \Psi\,. \tag{2}$$

Here, $g^{\mu\nu}$ is the contravariant metric tensor, g its determinant, m is the nucleon mass, Ψ its wave function. The Schwarzschild metric tensor is the most common used, but its spatial part has singularity at $r = r_s$, where r_s is the Schwarzschild radius. To remove this singularitywe use Eddington-Filkenstein metric tensor.

$$g^{\mu\nu} = \begin{pmatrix} 1+\frac{r_s}{r} & -\frac{r_s}{r} & 0 & 0 \\ -\frac{r_s}{r} & -(1-\frac{r_s}{r}) & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{r^2 \sin^2\theta} \end{pmatrix} \tag{3}$$

Here, r_s is the Schwarzschild radius, writing down the Klein-Gordon equation in the Eddington-Filkenstein metrics and taking nonrelativistic limit we get the Schrödinger equation in the curved space.

$$\psi(t,\mathbf{r},\theta,\varphi) = \mathrm{e}^{-i\frac{E}{\hbar}t}\, \psi(\mathbf{r},\theta,\varphi) \tag{4}$$

After that using the standard transformation, we derive the stationary Schrödinger equation in a curved space created by the MBH

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2}+\widetilde{V}_1(r)+\widetilde{V}_2(r)+V^N(r)+\frac{l(l+1)}{2\mu r^2}\right]U_l(k,r)=E_pU_l(k,r), \quad (5)$$

where $\psi_l(k,r)=\dfrac{U_l(k,r)}{kr}$, k is the nucleon momentum,

$$V_N=-\frac{GMm}{r},$$
$$\widetilde{V}_1(r)=i(\hbar c)r_s\left[-\frac{1}{2r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right], \quad (6)$$
$$\widetilde{V}_2(r)=\frac{1}{2}\frac{(\hbar c)^2}{mc^2}\frac{r_s}{r}\left[\frac{1}{r^2}-\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial r^2}\right].$$

Due to the imaginary potential $\widetilde{V}_1(r)$, which is generated by a curved space, the absorption cross section of nucleons is non-zero and can be written as

$$\sigma=i\frac{4m}{\hbar^2 k}\left(4\pi\right)^2\sum_{l=0}^{\infty}\frac{2l+1}{4\pi}\int_{r>r_s}^{\infty}dr\cdot r^2\frac{U_l(k,r)}{kr}\left(-i\hbar\left[\frac{cr_s}{r}\frac{\partial}{\partial r}+\frac{r_s c}{2r^2}\right]\frac{U_l(k,r)}{kr}\right). \quad (7)$$

Using Eq. (7) we calculated the mass gain by the MBH when it passes through sun.